A New Spatial Multiple Discrete–Continuous Modeling Approach to Land Use Change Analysis

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Presentation Overview

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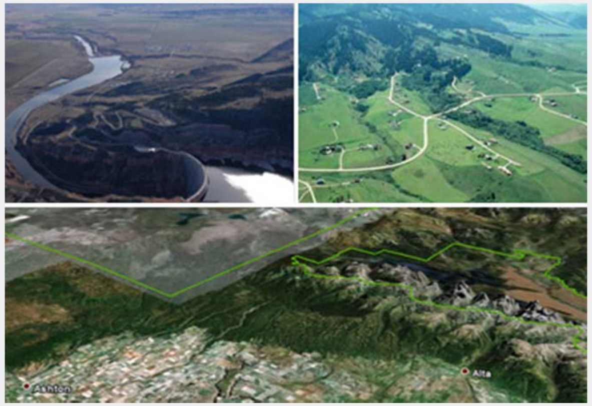
Introduction

Land Use Modeling

- Land-use models are used in many fields
  - Planning,
  - Urban science,
  - Ecological science,
  - Climate science,
  - Geography,
  - Watershed hydrology,
  - Environmental science,
  - Political science, and
  - Transportation

Ottawa Downtown: Taken from www.dreamstime.com



Taken from www.greateryellowstone.com

Why is Land Use Modeling Important?

- Used to examine future land-use scenarios
- Evaluate potential effects of policies
- Recently, substantial attention on
  - Biodiversity loss,
  - Deforestation consequences, and
  - Carbon emissions increases caused by land-use development
- Land-use patterns constitute one of the most important “habitat” elements characterizing Earth’s terrestrial and aquatic ecosystems

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Objectives

Objectives

- Develop new econometric approach to specify and estimate a land-use change model
    - Capable of predicting both the type and intensity of urban development patterns over large geographic areas
    - Explicitly acknowledges geographic proximity-based spatial dependencies in these patterns

Methodological Perspective

- Specification and estimation of a spatial multiple discrete-continuous probit (MDCP) model
- Allows the dependent variable to exist in multiple discrete states with an intensity associated with each discrete state
- Accommodates
  - Spatial dependencies,
  - Spatial heterogeneity,
  - Heteroscedasticity, in the dependent variable
- Applicable where social and spatial dependencies between decision agents (or observation units) lead to spillover effects in multiple discrete-continuous choices (or states)

Empirical Perspective

- Model land-use in multiple discrete states
- Along with the area invested in each land-use discrete state, within each spatial unit in an entire urban region
- Hybrid of three different strands of model types (pattern, process and spatial-based models) used in the land-use analysis literature

Empirical Context

Earlier Literature

- Three Modeling Approaches
  - Pattern-based Models
  - Process-based Models
  - Spatial-based Models

Pattern Based Models

- Developed by geographers and natural scientists
- Well suited for land-use modeling over relatively large geographic extents (such as urban regions or entire states or even countries)
- Unit of analysis: Aggregated Spatial Unit (Large grid, TAZ, Census Tract, County or State)
- Two types
  - Cellular automata-based Models
  - Empirical models at aggregated spatial unit level

Cellular Automata-based Models

- Hypothesizes the nature of the deterministic or probabilistic updating functions
- Simulates the states of cells over many “virtual” time periods,
- Aggregates up the states of the cells at the end to obtain land-use patterns
- Limitations
    - Updating functions not based on actual data → no direct evidence linking the updating mechanism at the cell level to the spatial evolution of land-use patterns at the aggregate spatial unit level
    - Do not use exogenous variables such as socio-demographic characteristics of spatial units, transportation network features, and other environmental features→ Policy value is extremely limited

Empirical models at Aggregated Spatial Unit Level

- Relates transportation network, pedoclimatic, biophysical and accessibility variables to land-use patterns
- Can be used in a simulation setting to predict land-use patterns in response to different exogenously imposed policy scenarios
- Not formulated in a manner that appropriately recognizes the multiple discrete-continuous nature of land-use patterns in the aggregated spatial units
- Do not adequately consider population characteristics of spatial units in explaining land-use patterns within that unit

Process-based Models

- Developed by economists
- Well suited for modeling landowners' decisions of land-use type choice for their parcels
- Unit of analysis: Land-owner is considered as an economic agent
- Considers the human element in land-use modeling
- Forward-looking inter-temporal land use decisions based on profit-maximizing behavior

Process-based Models

- Difficulties incorporating spatial considerations at this micro-level
- High data and computing demands when analysis is being conducted at the level of entire urban regions or states in a country
- Presence of land-use and zoning regulations → Individual landowners may not have carte blanche authority
- Multiple parcels under the purview of a single decision-making agent → Multiple parcels in close proximity tend to get similarly developed

Spatial-based models

- Emphasis on spatial dependence among spatial units (in pattern-based models) or among landowners (in process-based models)

- Caused by diffusion effects, or zoning and land-use regulation effects, or social interaction effects, or observed and unobserved location-related influences

- Two most dominant spatial formulations → Spatial lag and spatial error formulations
- Spatial lag structure
  - Considers spillover effects caused by observed exogenous variables at one spatial location influencing land-use patterns in adjacent locations
  - Generates spatial heteoscedasticity.

Spatial-based models

- Spatial heterogeneity → Differences in relationships between the dependent variable and the independent variables across decision-makers or spatial units in a study region

- Essential to accommodate local variations (*i.e.*, recognize spatial non-stationarity) in the relationship across a study region rather than settle for a single global relationship

Econometric Considerations

Past Studies

- In the past decade , much emphasis has been placed on accommodating spatial correlation in binary/ordered models (spatial regression models, weighted geographic regression, spatial probit and spatial tobit).

- Estimation has mostly been done using simulation techniques (GHK and Bayesian MCMC).

- Standard RIS and MCMC-based simulators are cumbersome to implement in typical empirical contexts

RECENT ADVANCES

- Spatial land use change model for unordered choice case, including spatial lag dependency, random heterogeneity, and general covariance matrix.

- A new estimation technique has been proposed (MACML, Bhat(2012)).

Transition

- Discrete choice field has moved forward from ordered/unordered cases to multiple discrete-continuous models.

- A realistic representation of choices made in real-life.

Multiple discrete-continuous choice models (MDC)

- Capable of accommodating multiple choices

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Model Formulation

Utility function

$$\max U_q(\boldsymbol{x_q}) = \sum_{k=1}^{K} \frac{\gamma_{qk}}{\alpha_{qk}} \psi_{qk} \left(\left(\frac{x_{qk}}{\gamma_{qk}} + 1 \right)^{\alpha_{qk}} - 1 \right)$$

$$s.t. \sum_{k=1}^{K} x_{qk} = E_q$$

$U(\boldsymbol{x}_q)$ is a quasi-concave, increasing, and continuously differentiable function with respect to the consumption quantity vector $\boldsymbol{x}$

α_{qk}, γ_{qk} and ψ_{qk} are parameters associated with alternative $\boldsymbol{k}$ for decision maker $\boldsymbol{q}$

x_{qk} is the consumption/investment value of outside alternative k

Utility function

- Role of ψ_{qk}

$$\frac{\partial U(\boldsymbol{x}_{qk})}{\partial x_{qk}} = \psi_{qk}\left(\frac{x_{qk}}{\gamma_{qk}}+1\right)^{\alpha_{qk}-1}$$

ψ_{qk} : baseline (at zero consumption/investment) marginal utility, should always be positive

x_{qk} : Investment/consumption value of an alternative k (inside good) by decision maker q

ψ_{qk} / ψ_{ql} : marginal rate of substitution at zero consumption

Higher baseline implies less likelihood of a corner solution for an alternative k

$$\psi_{qk} = \exp(\tilde{\boldsymbol{z}}_{\boldsymbol{qk}}, \xi_{qk}) = \exp(\boldsymbol{\beta}'_{\boldsymbol{q}}\tilde{\boldsymbol{z}}_{\boldsymbol{qk}} + \xi_{qk}) \text{ or } \overline{\psi}^{*}_{qk} = \ln(\psi_{qk}) = \boldsymbol{\beta}'_{\boldsymbol{q}}\tilde{\boldsymbol{z}}_{\boldsymbol{qk}} + \xi_{qk},$$

Utility function

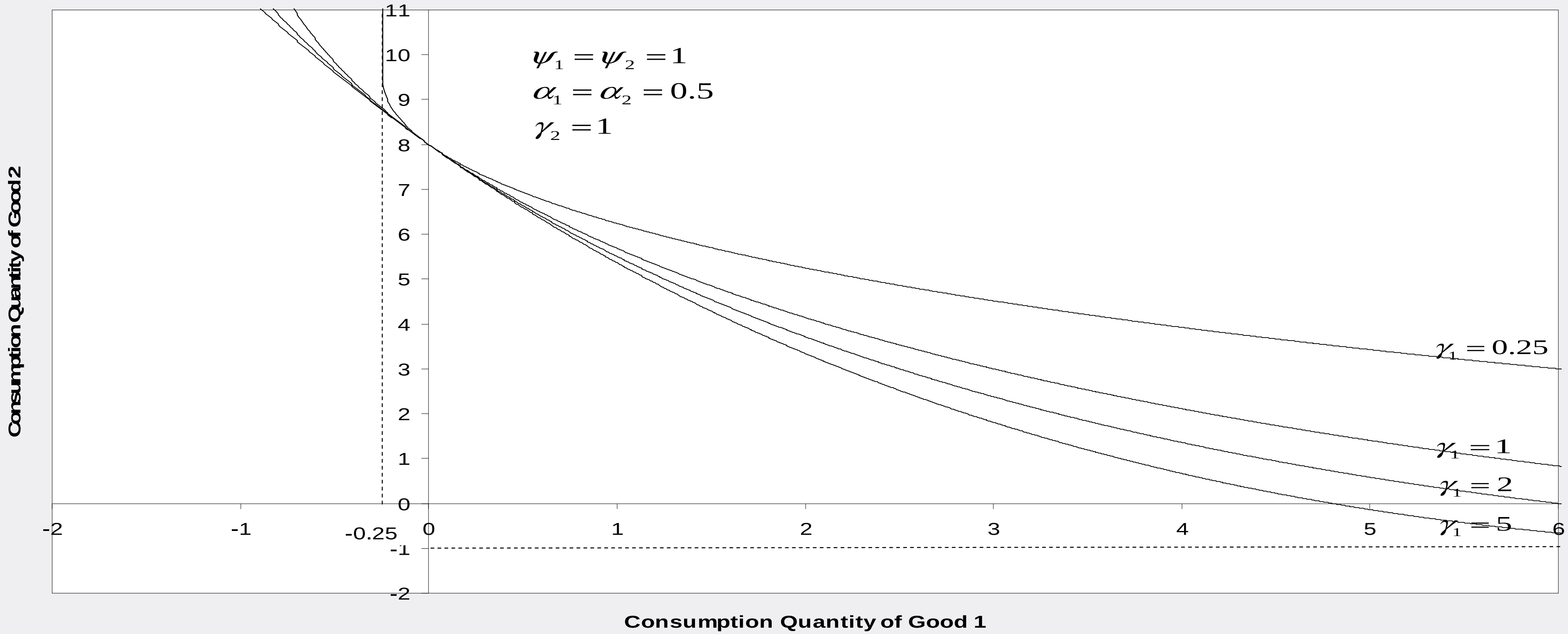
- Role of $\gamma_{qk} (\gamma_{qk} > 0)$

$$\text{Slope}(x_{q1}, x_{q2}) = \frac{\partial U(\boldsymbol{x_q}) / \partial x_{q1}}{\partial U(\boldsymbol{x_q}) / \partial x_{q2}} = \frac{\left(\dfrac{x_{q2}}{\gamma_{q2}} + 1\right)^{1-\alpha_{q2}}}{\left(\dfrac{x_{q1}}{\gamma_{q1}} + 1\right)^{1-\alpha_{q1}}} \times \frac{\psi(x_{q1})}{\psi(x_{q2})}$$

At $x_{q1} = -\gamma_{q1}$, slope $= \infty$

At $x_{q2} = -\gamma_{q2}$, slope $= 0$

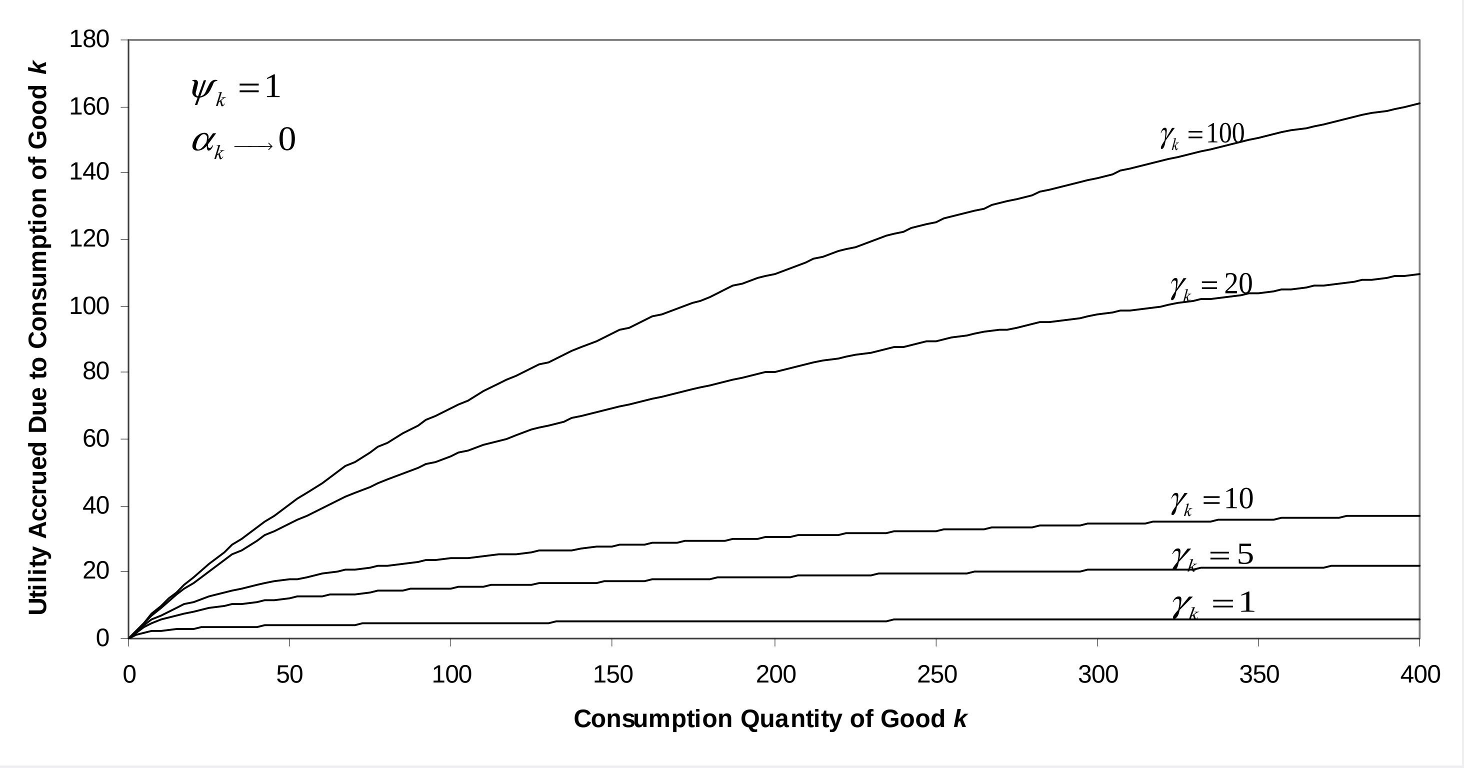
- Indifference Curves



Indifference Curves Corresponding to Different Values of γ_1

Utility function

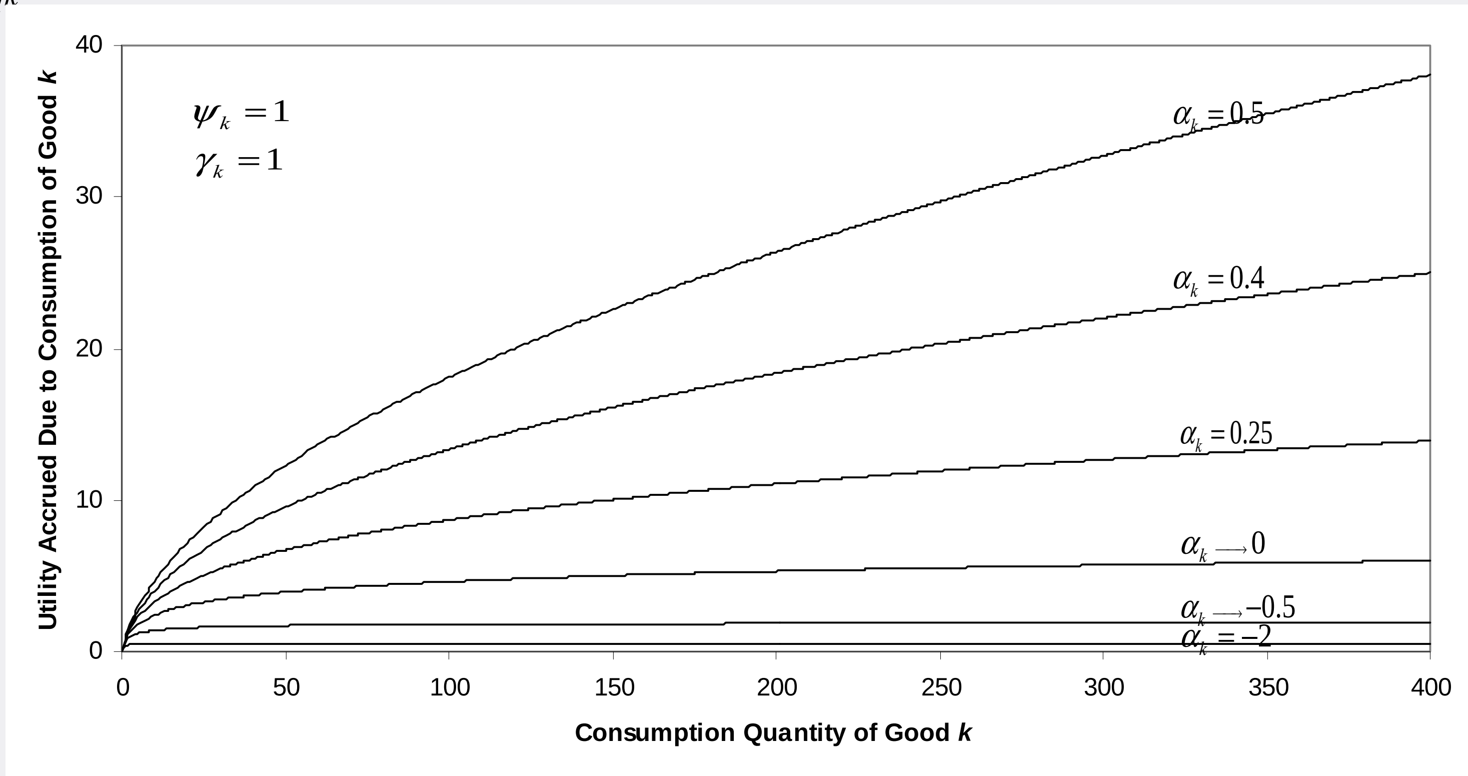
- Role of $\gamma_{qk} (\gamma_{qk} > 0)$



Effect of γ_{qk} Value on Good *k*'s Subutility Function Profile

Utility function

- Role of α_{qk}



Effect of α_{qk} Value on Good *k*'s Subutility Function Profile

KKT first order condition (MDCP)

Since only differences in the logarithm of the baseline utilities matter, we subtract the logarithm of baseline utility of outside alternative from **k**-1 inside alternatives and normalize the logarithm of the baseline utility for the outside alternative to zero. Baseline utility is parameterized to ensure positive value of baseline utility

Baseline Utility

$$\overline{\psi}_{qk} = \ln(\overline{\psi}^{*}_{qk}) - \ln(\overline{\psi}^{*}_{qK}) = \boldsymbol{\beta}'_{\boldsymbol{q}}(\tilde{\boldsymbol{z}}_{\boldsymbol{qk}} - \tilde{\boldsymbol{z}}_{\boldsymbol{qK}}) + (\xi_{qk} - \xi_{qK})$$

$$= \boldsymbol{\beta}'_{\boldsymbol{q}} \boldsymbol{z}_{\boldsymbol{qk}} + \varepsilon_{qk}, \ \boldsymbol{z}_{\boldsymbol{qk}} = \tilde{\boldsymbol{z}}_{\boldsymbol{qk}} - \tilde{\boldsymbol{z}}_{\boldsymbol{qK}}, \varepsilon_{qk} = (\xi_{qk} - \xi_{qK}) \ \forall k \neq K$$

$$\overline{\psi}_{qK} = \ln(\overline{\psi}^{*}_{qK}) - \ln(\overline{\psi}^{*}_{qK}) = 0 \ \ for \ \ k = K.$$

$$\psi_{qk} = \exp(\overline{\psi}_{qk})$$

$$\boldsymbol{\beta}_q \sim MVN_D(\boldsymbol{b}, \boldsymbol{\Omega})$$

$$\boldsymbol{\beta}_q = \boldsymbol{b} + \tilde{\boldsymbol{\beta}}_q$$

$$\tilde{\boldsymbol{\beta}}_q \sim MVN_D(\boldsymbol{0}_D, \boldsymbol{\Omega})$$

$\boldsymbol{b}$: mean estimate of $\boldsymbol{\beta}_q$

$\boldsymbol{\Omega}$: Covariance matrix for random coefficient

$\tilde{\boldsymbol{\beta}}_q$: random coefficient standard deviation with mean zero and covariance $\boldsymbol{\Omega}$

KKT first order condition (MDCP) cont..

KKT first order conditions

$$\exp(\boldsymbol{b}'\boldsymbol{z}_{qk} + \tilde{\boldsymbol{\beta}}'_q\boldsymbol{z}_{qk} + \varepsilon_{qk})\left(\frac{x^*_{qk}}{\gamma_k}+1\right)^{\alpha_k-1} - \lambda_q = 0 \quad \text{,if} \quad x^*_{qk} > 0, \quad k = 1, 2, ..., K-1$$

$$\exp(\boldsymbol{b}'\boldsymbol{z}_{qk} + \tilde{\boldsymbol{\beta}}'_q\boldsymbol{z}_{qk} + \varepsilon_{qk})\left(\frac{x^*_{qk}}{\gamma_k}+1\right)^{\alpha_k-1} - \lambda_q < 0 \quad \text{,if} \quad x^*_{qk} = 0, \quad k = 1, 2, ..., K-1$$

where $\lambda_q = \left(x^*_{qK} + \gamma_K\right)^{\alpha_K-1}$ x^*_{qK} is the investment value corresponds to outside alternative K

Final KKT first order expression

$$y^*_{qk} = (V_{qk} - V_{qK}) + \tilde{\varepsilon}_{qk} = 0 \quad \text{,if} \quad x^*_{qk} > 0, \quad k = 1, 2, ..., K-1$$

$$y^*_{qk} = (V_{qk} - V_{qK}) + \tilde{\varepsilon}_{qk} < 0 \quad \text{,if} \quad x^*_{qk} = 0, \quad k = 1, 2, ..., K-1$$

where $V_{qk} = \boldsymbol{b}'\boldsymbol{z}_{qk} + (\alpha_k - 1)\ln\left(\frac{x^*_{qk}}{\gamma_k}+1\right)$ for $k = 1, 2, ..., K-1$

$$V_{qK} = (\alpha_K - 1)\ln\left(x^*_{qK} + \gamma_K\right), \qquad \tilde{\varepsilon}_{qk} = \tilde{\boldsymbol{\beta}}'_q\boldsymbol{z}_{qk} + \varepsilon_{qk}$$

V_{qk} : utility of the alternative k

$\tilde{\varepsilon}_{qk}$: difference in the error between alternative k and outside alternative K

KKT first order condition (SMDCP)

We introduce the spatial auto-correlation through baseline utility as follow:

$$\bar{\psi}_{qk} = \boldsymbol{\beta}_q' \boldsymbol{z}_{qk} + \varepsilon_{qk} + \delta_k \sum_{q'} w_{qq'} \bar{\psi}_{q'k}, \quad for \;\; k = 1, 2, ..., K-1$$

$$\bar{\psi}_{qK} = 0 \text{ for } k = K.$$

Following the steps of MDCP model, we can see that difference in utility is distributed with mean $\boldsymbol{B}$ and covariance $\boldsymbol{\Sigma}$

$$\mathbf{y}^* \sim MVN_{Q\times(K-1)}(\boldsymbol{B}, \boldsymbol{\Sigma})$$

Where $\boldsymbol{B_q} = (V_{q1} - V_{qK}, V_{q2} - V_{qK}, ..., V_{q,K-1} - V_{qK})'$ $[(K-1)\times 1 \text{ vector}]$

$$V_{qk} = [\mathbf{Szb}]_{d_{qk}} + (\alpha_k - 1)\ln\left(\frac{x^*_{qk}}{\gamma_k} + 1\right) \quad \text{for} \quad k = 1, 2, ..., K-1$$

$$\boldsymbol{\Sigma} = \boldsymbol{S}[\tilde{\boldsymbol{\Lambda}} + \tilde{\boldsymbol{\Omega}}]\boldsymbol{S}' \quad [Q(K-1)\times Q(K-1) \text{ matrix}]$$

$$\tilde{\boldsymbol{\Omega}} = \vec{z}(\mathbf{IDEN_Q} \otimes \boldsymbol{\Omega})\vec{z}' \quad [Q(K-1)\times Q(K-1) \text{ matrix}]$$

$$\boldsymbol{B} = (\boldsymbol{B}_1', \boldsymbol{B}_2', ..., \boldsymbol{B}_Q')' \quad [Q(K-1)\times 1 \text{ vector}]$$

$$V_{qK} = (\alpha_K - 1)\ln\left(x^*_{qK} + \gamma_K\right) \quad \text{for} \quad k = K$$

$$\tilde{\boldsymbol{\Lambda}} = \mathbf{IDEN_Q} \otimes \boldsymbol{\Lambda} \quad [Q(K-1)\times Q(K-1) \text{ matrix}]$$

$\tilde{\boldsymbol{\Omega}}$ and $\tilde{\boldsymbol{\Lambda}}$ are the random coefficient covariance matrix and differenced error covariance matrix, respectively

$$\vec{\boldsymbol{z}} = \begin{bmatrix} \boldsymbol{z_1} & 0 & 0 & \dots & 0 \\ 0 & \boldsymbol{z_2} & 0 & \dots & 0 \\ 0 & 0 & \boldsymbol{z_3} & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & \boldsymbol{z_Q} \end{bmatrix} \; [Q(K-1)\times QD \text{ matrix}]$$

$$\boldsymbol{\delta} = \begin{bmatrix} \delta_1 & 0 & 0 & \dots & 0 \\ 0 & \delta_2 & 0 & \dots & 0 \\ 0 & 0 & \delta_3 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & \delta_{K-1} \end{bmatrix} [(K-1)\times(K-1) \text{ matrix}]$$

$$\mathbf{W} = (\boldsymbol{1_{QQ}} \otimes \boldsymbol{\delta}).*(\tilde{\boldsymbol{W}} \otimes \boldsymbol{IDEN_{K\text{-}1}})$$

$\tilde{\boldsymbol{W}}$ is a $(Q\times Q)$ weight matrix with weight $w_{qq'}$ as its elements

$$\mathbf{S} = \left[\mathbf{IDEN}_{Q(K-1)} - \mathbf{W}\right]^{-1} \; [Q(K-1)\times Q(K-1) \text{ matrix}]$$

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Model Estimation

Model Estimation

Partition the vector $\mathbf{y}^*$ into two sub-vector to represent chosen and non-chosen alternatives.

$$\tilde{\boldsymbol{y}}^* = \left([\tilde{\boldsymbol{y}}^*_{NC}]', [\tilde{\boldsymbol{y}}^*_{C}]' \right)'$$

We can retrieve $\tilde{\boldsymbol{y}}^*$ from $\mathbf{y}^*$ as follow: $\tilde{\boldsymbol{y}}^* = \mathbf{R}\mathbf{y}^*$

Where, $\mathbf{R}$ is a re-arrangement matrix of dimension with zeros and ones

For example, consider the case of three grids and five land-use alternatives. The last alternative is the "undeveloped" land-use state, which is the outside alternative. Among the remaining four alternatives, let grid 1 be invested in alternatives 1 and 4 (not invested in alternatives 2 and 3), let grid 2 be invested in alternatives 2 and 3 (not invested in alternatives 1 and 4), and let grid 3 be invested in alternative 1 (not invested in alternatives 2, 3, and 4). Then, the re-arrangement $\mathbf{R}$ matrix is:

Model Estimation Cont..

$$\mathbf{R} == \left[\begin{array}{cccc|cccc|cccc}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
\hline
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0
\end{array}\right] = \left[\begin{array}{c} \mathbf{R}_{NC} \\ \hline \mathbf{R}_{C} \end{array}\right]$$

With further re-arrangement, we can write the likelihood function as follow:

$$L_{ML}(\boldsymbol{\theta}) = \text{Prob}(\boldsymbol{x}^{*}) = \det(\mathbf{J}) \int_{\text{h}_{NC}=-\infty}^{0} f_{Q(K-1)}\left(\text{h}_{NC}, 0_{L_C} \mid \tilde{\boldsymbol{B}}, \tilde{\boldsymbol{\Sigma}}\right) d\text{h}_{NC},$$

Where
$$\det(\mathbf{J}) = \prod_{q=1}^{Q} \left[\left\{ \prod_{k \in \bar{L}_{qC}} \frac{1-\alpha_k}{x_{qk}^{*} + \gamma_k} \right\} \left\{ \sum_{k \in \bar{L}_{qC}} \left(\frac{x_{qk}^{*} + \gamma_k}{1-\alpha_k} \right) \right\} \right]$$

$\mathbf{J}$: Jacobian

h_{NC}: Number of non-chosen alternatives

$\tilde{\boldsymbol{B}}$ and $\tilde{\boldsymbol{\Sigma}}$ are the mean and covariance matrix of utility difference

Model Estimation Cont..

- The likelihood function involves integration of dimension equal to number of non-chosen alternatives
- Very high dimensional integration
- Traditional simulation techniques such as Bayesian inference method and Maximum simulated likelihood are not suitable
- We use Bhat's Maximum Approximate Composite Marginal Likelihood (MACML) inference approach.

CML function

$$L_{CML}(\boldsymbol{\theta}) = \text{Prob}(\boldsymbol{x}_q^*, \boldsymbol{x}_{q'}^*)$$

$$= \prod_{q=1}^{Q-1} \prod_{q'=q+1}^{Q} \det(\mathbf{J}_{qq'}) \times \left(\overline{\boldsymbol{\omega}}_{\tilde{\Sigma}_{qq',C}}\right)^{-1} \left[\phi_{L_{qq',C}}\left(\tilde{\boldsymbol{B}}^*_{\boldsymbol{qq',C}}, \tilde{\boldsymbol{\Sigma}}^*{}_{qq',C}\right)\right] \times \left[\Phi_{L_{qq',NC}}\left(\breve{\boldsymbol{B}}^*_{\boldsymbol{qq',NC}}, \breve{\boldsymbol{\Sigma}}^*{}_{qq',NC}\right)\right]$$

Where $\boldsymbol{\omega}_{\breve{\Sigma}_{qq',NC}}$ is the diagonal matrix of standard deviation of $\breve{\Sigma}_{qq',NC}$

$$\det(\mathbf{J}_{qq'}) = \prod_{l=q,q'} \left[\left\{ \prod_{k \in \bar{L}_{lC}} \frac{1-\alpha_k}{x_{lk}^* + \gamma_k} \right\} \left\{ \sum_{k \in \bar{L}_{lC}} \left(\frac{x_{lk}^* + \gamma_k}{1-\alpha_k} \right) \right\} \right]$$

$$\tilde{\boldsymbol{B}}^*_{\boldsymbol{qq',C}} = \boldsymbol{\omega}^{-1}_{\tilde{\Sigma}_{qq',C}} \left(-\tilde{\boldsymbol{B}}_{\boldsymbol{qq',C}}\right)$$

$$\tilde{\boldsymbol{\Sigma}}^*{}_{qq',C} = \boldsymbol{\omega}^{-1}_{\tilde{\Sigma}_{qq',C}} \tilde{\boldsymbol{\Sigma}}_{qq',C} \boldsymbol{\omega}^{-1}_{\tilde{\Sigma}_{qq',C}}$$

$$\breve{\boldsymbol{\Sigma}}^*{}_{qq',NC} = \boldsymbol{\omega}^{-1}_{\breve{\Sigma}_{qq',NC}} \breve{\boldsymbol{\Sigma}}_{qq',NC} \boldsymbol{\omega}^{-1}_{\breve{\Sigma}_{qq',NC}}$$

$$\breve{\boldsymbol{\Sigma}}_{qq',NC} = \tilde{\boldsymbol{\Sigma}}_{qq',NC} - \tilde{\boldsymbol{\Sigma}}'_{qq',NC,C} (\tilde{\boldsymbol{\Sigma}}_{qq',C})^{-1} \tilde{\boldsymbol{\Sigma}}_{qq',NC,C}$$

$$\breve{\boldsymbol{B}}^*_{\boldsymbol{qq',NC}} = \boldsymbol{\omega}^{-1}_{\breve{\Sigma}_{qq',NC}} \left(-\breve{\boldsymbol{B}}_{\boldsymbol{qq',NC}}\right),$$

$$\breve{\boldsymbol{B}}_{\boldsymbol{qq',NC}} = \tilde{\boldsymbol{B}}_{\boldsymbol{qq',NC}} + \tilde{\boldsymbol{\Sigma}}'_{qq',NC,C} (\tilde{\boldsymbol{\Sigma}}_{qq',C})^{-1} (-\tilde{\boldsymbol{B}}_{\boldsymbol{qq',C}}),$$

Simulation

- Use MACML (Bhat, 2011)
- 4 alternatives, 3 coefficients: 1 fixed, 2 random
- Two sets of spatial auto-correlation parameters
- 2000 observations, 30 datasets with 10 permutation (a total of 300 runs)
- gamma profile
- Comparison with additional restrictive models (spatial IID MDCP, spatial homogeneous MDCP and MDCP). Single permutation is used in comparison due to low approximation error

Simulation Cont..

$\boldsymbol{\beta}_q \sim MVN_D(\boldsymbol{b}, \boldsymbol{\Omega})$ b = (0.5, –1, 1)

$$\boldsymbol{\Omega} = \begin{bmatrix} 0.81 & 0.54 \\ 0.54 & 1.00 \end{bmatrix} = \mathbf{L}_{\Omega}\mathbf{L}'_{\Omega} = \begin{bmatrix} 0.90 & 0.00 \\ 0.60 & 0.80 \end{bmatrix} \begin{bmatrix} 0.90 & 0.60 \\ 0.00 & 0.80 \end{bmatrix}$$

$\boldsymbol{\xi}_q \sim MVN_K(\boldsymbol{0}_K, \boldsymbol{\Lambda})$

$\boldsymbol{\varepsilon}_{qk} = \boldsymbol{\xi}_{qk} - \boldsymbol{\xi}_{q1}$

$$\boldsymbol{\Lambda} = \begin{bmatrix} 1.00 & 0.50+0.20 & 0.50+0.40 \\ 0.50+0.20 & 0.50+0.80 & 0.50+0.31 \\ 0.50+0.40 & 0.50+0.31 & 0.50+0.99 \end{bmatrix} = \begin{bmatrix} 1.00 & 0.70 & 0.90 \\ 0.70 & 1.30 & 0.81 \\ 0.90 & 0.81 & 1.49 \end{bmatrix}$$

$$= \mathbf{L}_{\Lambda}\mathbf{L}'_{\Lambda} = \begin{bmatrix} 1.00 & 0.00 & 0.00 \\ 0.70 & 0.90 & 0.00 \\ 0.90 & 0.20 & 0.80 \end{bmatrix} \begin{bmatrix} 1.00 & 0.70 & 0.90 \\ 0.00 & 0.90 & 0.20 \\ 0.00 & 0.00 & 0.80 \end{bmatrix}$$

$(\delta_1 = 0.1,\ \delta_2 = 0.2,\ \delta_3 = 0.3)$ $(\delta_1 = 0.6,\ \delta_2 = 0.7,\ \delta_3 = 0.8)$ $(\gamma_1 = 1,\ \gamma_2 = 1,\ \gamma_3 = 1,\ \gamma_4 = 0)$

All the notations are same as mentioned before

Simulation results (Low spatial dependency case)

Parameter	True Value	Parameter Estimates			Standard Error Estimates			
		Mean Est.	Abs. Bias	Absolute Percentage Bias (APB)	Finite Sample St. Err. (FSSE)	Asymptotic St. Err. (ASE)	Relative Efficiency	Approximation error (APERR)
b_1	0.5	0.48	0.02	4.00	0.024	0.030	1.25	0.001722
b_2	-1.0	-1.02	0.02	2.00	0.029	0.028	0.97	0.001781
b_3	1.0	0.99	0.01	1.00	0.023	0.024	1.04	0.001225
$l_{\Omega 1}$	0.9	0.86	0.04	4.44	0.024	0.021	0.88	0.002232
$l_{\Omega 2}$	0.6	0.58	0.02	3.33	0.024	0.029	1.21	0.001310
$l_{\Omega 3}$	0.8	0.78	0.02	2.50	0.028	0.031	1.11	0.001480
γ_1	1.0	0.98	0.02	2.00	0.038	0.038	1.00	0.003031
γ_2	1.0	0.97	0.03	3.00	0.048	0.039	0.82	0.003029
γ_3	1.0	0.96	0.04	4.00	0.049	0.042	0.86	0.003965
$l_{\Lambda 1}$	0.7	0.70	0.00	0.00	0.025	0.019	0.76	0.001797
$l_{\Lambda 2}$	0.9	0.91	0.01	1.11	0.023	0.016	0.70	0.001309
$l_{\Lambda 3}$	0.9	0.90	0.00	0.00	0.021	0.018	0.86	0.002493
$l_{\Lambda 4}$	0.2	0.21	0.01	5.00	0.014	0.016	1.14	0.002852
$l_{\Lambda 5}$	0.8	0.80	0.00	0.00	0.016	0.012	0.75	0.002362
δ_1	0.1	0.10	0.00	0.00	0.005	0.004	0.80	0.000065
δ_2	0.2	0.20	0.00	0.00	0.008	0.006	0.75	0.000175
δ_3	0.3	0.30	0.00	0.00	0.011	0.008	0.73	0.000324
Overall mean value across parameters			0.01	1.90	0.024	0.022	0.92	0.001832

Simulation results (High spatial dependency case)

Parameter	True Value	Parameter Estimates			Standard Error Estimates			
		Mean Est.	Abs. Bias	Absolute Percentage Bias (APB)	Finite Sample St. Err. (FSSE)	Asymptotic St. Err. (ASE)	Relative Efficiency	Approximation error (APERR)
b_1	0.5	0.48	0.02	4.00	0.041	0.052	1.27	0.000943
b_2	-1.0	-1.04	0.04	4.00	0.038	0.047	1.24	0.000792
b_3	1.0	0.98	0.02	2.00	0.022	0.028	1.27	0.000704
$l_{\Omega 1}$	0.9	0.87	0.03	3.33	0.019	0.023	1.21	0.000866
$l_{\Omega 2}$	0.6	0.58	0.02	3.33	0.053	0.047	0.89	0.001881
$l_{\Omega 3}$	0.8	0.80	0.00	0.00	0.041	0.046	1.12	0.001093
γ_1	1.0	0.94	0.06	6.00	0.081	0.082	1.01	0.002657
γ_2	1.0	0.96	0.04	4.00	0.085	0.081	0.95	0.001008
γ_3	1.0	0.89	0.11	11.00	0.070	0.054	0.77	0.000640
$l_{\Lambda 1}$	0.7	0.71	0.01	1.43	0.017	0.017	1.00	0.001736
$l_{\Lambda 2}$	0.9	0.90	0.00	0.00	0.009	0.012	1.33	0.002966
$l_{\Lambda 3}$	0.9	0.89	0.01	1.11	0.020	0.018	0.90	0.002270
$l_{\Lambda 4}$	0.2	0.19	0.01	5.00	0.037	0.029	0.78	0.002260
$l_{\Lambda 5}$	0.8	0.83	0.03	3.75	0.019	0.015	0.79	0.001317
δ_1	0.6	0.60	0.00	0.00	0.048	0.037	0.77	0.000842
δ_2	0.7	0.69	0.01	1.43	0.109	0.105	0.96	0.001897
δ_3	0.8	0.74	0.06	7.50	0.110	0.129	1.17	0.005074
Overall mean value across parameters			0.03	3.40	0.048	0.049	1.03	0.001703

Comparison with restrictive models

- Spatial IID MDCP

$$\Lambda = \begin{bmatrix} 1.00 & 0.50 & 0.50 \\ 0.50 & 1.00 & 0.50 \\ 0.50 & 0.50 & 1.00 \end{bmatrix}$$

$$= \mathbf{L}_\Lambda \mathbf{L}'_\Lambda = \begin{bmatrix} 1.000 & 0.000 & 0.000 \\ 0.500 & 0.866 & 0.000 \\ 0.500 & 0.287 & 0.816 \end{bmatrix} \begin{bmatrix} 1.000 & 0.500 & 0.500 \\ 0.000 & 0.866 & 0.287 \\ 0.000 & 0.000 & 0.816 \end{bmatrix}$$

- Spatial homogeneous MDCP

  All the elements of random coefficient matrix is zero

- MDCP

  spatial auto-correlation parameters are zero

Comparison with restrictive models

Parameters	True Value	SIMDCP*		SHMDCP+		MDCP#	
		Mean Est.	Absolute percentage Bias (APB)	Mean Est.	Absolute percentage Bias (APB)	Mean Est.	Absolute percentage Bias (APB)
b_1	0.5	0.42	16.00	0.36	28.00	0.48	4.00
b_2	-1.0	-1.07	7.00	-1.02	2.00	-1.01	1.00
b_3	1.0	0.98	2.00	0.88	12.00	1.01	1.00
$l_{\Omega 1}$	0.9	0.89	1.11	-[a]	-	0.89	1.11
$l_{\Omega 2}$	0.6	0.63	5.00	-	-	0.57	5.00
$l_{\Omega 3}$	0.8	0.79	1.25	-	-	0.82	2.50
γ_1	1.0	0.85	15.00	0.73	27.00	0.66	34.00
γ_2	1.0	0.81	19.00	0.67	33.00	0.49	51.00
γ_3	1.0	0.58	42.00	0.26	74.00	0.24	76.00
$l_{\Lambda 1}$	0.7	-	-	0.85	21.43	0.69	1.43
$l_{\Lambda 2}$	0.9	-	-	1.25	38.89	0.91	1.11
$l_{\Lambda 3}$	0.9	-	-	0.99	10.00	0.90	0.00
$l_{\Lambda 4}$	0.2	-	-	0.32	60.00	0.21	5.00
$l_{\Lambda 5}$	0.8	-	-	1.20	50.00	0.85	6.25
δ_1	0.6	0.58	3.33	0.96	60.00	-	-
δ_2	0.7	0.71	1.43	0.80	14.29	-	-
δ_3	0.8	0.78	2.50	0.64	20.00	-	-
Overall mean value across parameters		0.09	9.64	0.24	32.19	0.13	13.53
Mean composite log-likelihood value at convergence		-123728.0236		-127060.8099		-124231.3780	
Number of times the adjusted composite likelihood ratio test (ADCLRT) statistic favors the SMDCP model[b]		All thirty times when compared with $\chi^2_{5,0.95} = 11.07$ value (mean ADCLRT statistic is 26.31)		All thirty times when compared with $\chi^2_{3,0.99} = 11.34$ value (mean ADCLRT statistic is 53.95)		All thirty times when compared with $\chi^2_{3,0.99} = 11.34$ value (mean ADCLRT statistic is 27.47)	

*SIMDCP: Spatial IID MDCP.
+SHMDCP*: Spatial homogeneous MDCP,
#MDCP: Aspatial MDCP.
The mean composite log-likelihood value for the high dependency SMDCP model at converged parameter is -122377.2998.

Inferences from simulation study

- Excellent recovery of parameters by MACML, irrespective of the magnitude of spatial dependence.

- Finite sample and asymptotic standard errors are also very close

- Ignoring error covariance, or spatial heterogeneity, or spatial dependence has serious impact on true parameter value

- Finally, Ignoring spatial heterogeneity is of much more serious consequence than ignoring error covariance effects or spatial lag dynamics

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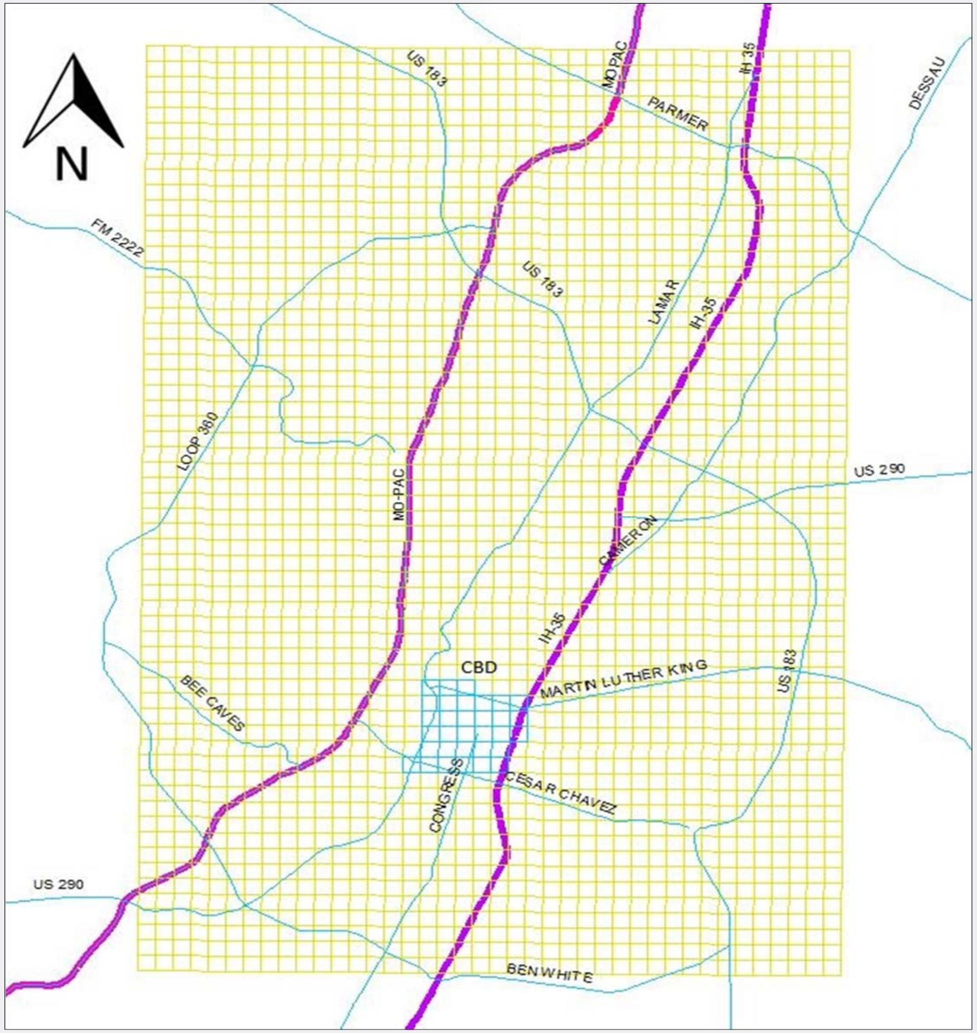
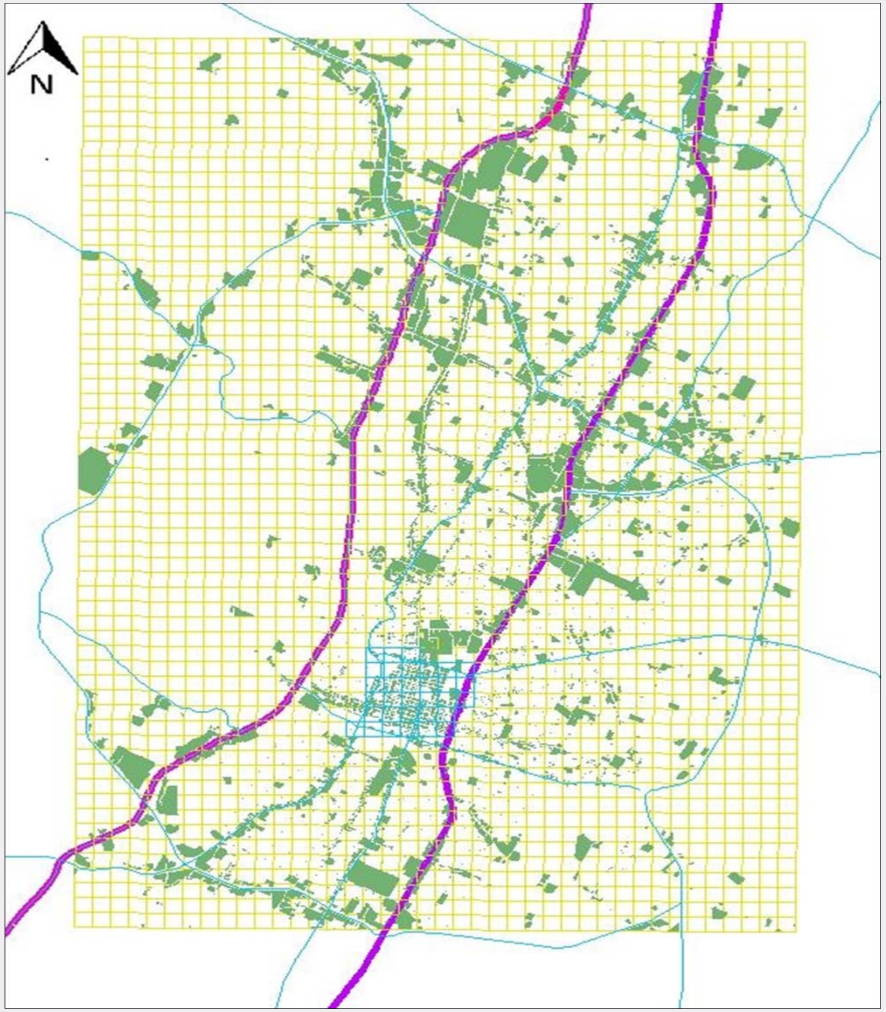
Empirical Application

Data and Variables

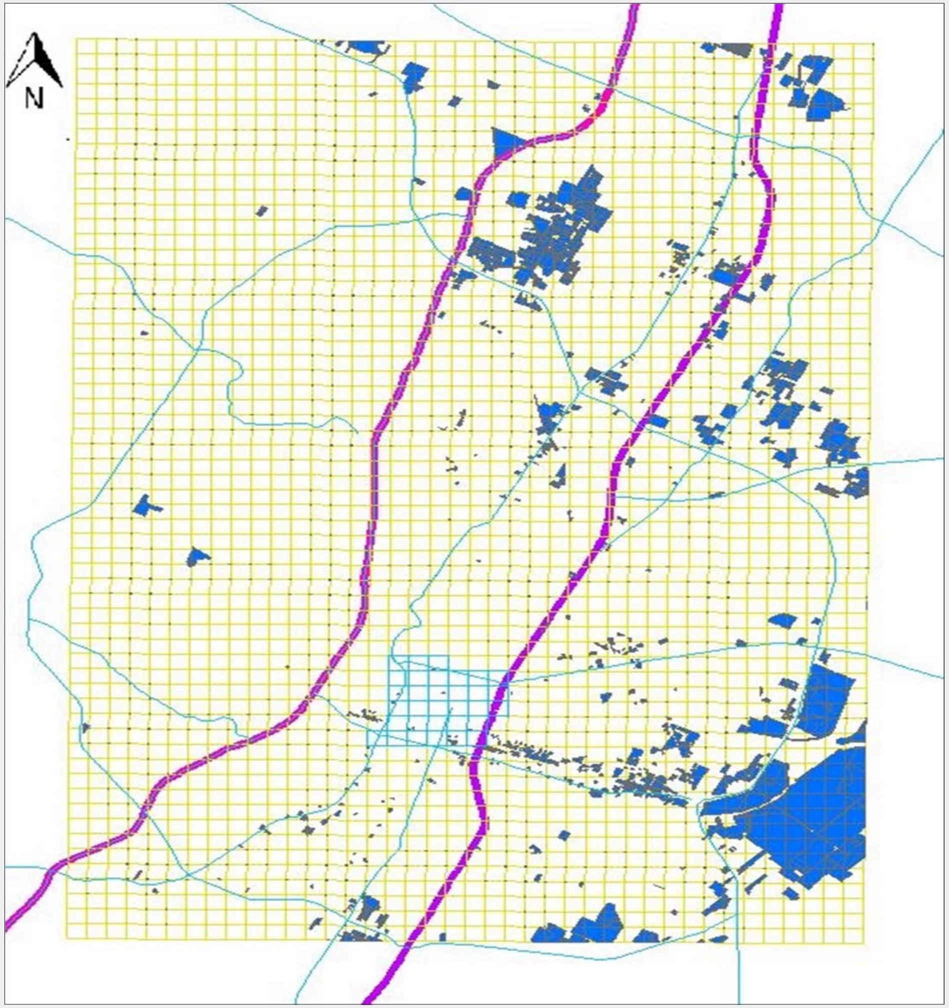
- Parcel level land-use inventory data for City of Austin, TX, year 2010.
- Land-use types were aggregated into commercial, industrial, residential and undeveloped (outside alternative).
- Size of analysis area : 145.91 sq miles.
- Size of analysis grid : 0.25 X 0.25 miles.
- Explanatory Variables : Road access measures (distance to highways and thoroughfares),
  distance to nearest school and hospital,
  fraction of area under floodplain,
  average elevation of the grid.
- Two Models were estimated and compared : MDCP and SMDCP



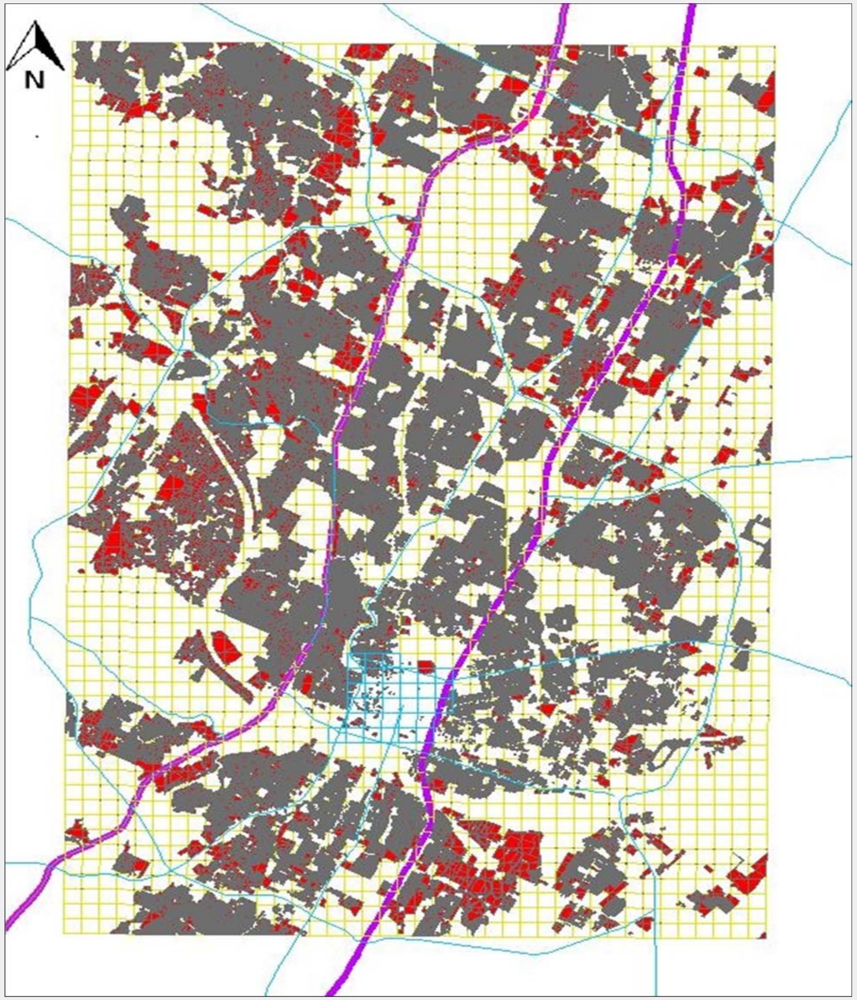
Study Area



Commercial land-use distribution



Industrial land-use distribution



Residential land-use distribution

Table 3a: Descriptive statistics of land-use type investment in the study area

Land-use type	Total number (%) of grids invested in land-use type[a]	Mean land-use area invested (sq mi)	Number of grids (% of total number) invested….	
			only in land-use type and the undeveloped land-use state	in other (inside) land-use types too
Commercial	1304 (55)	0.0136	103 (8)	1201 (92)
Industrial	579 (24)	0.0134	52 (9)	527 (91)
Residential	1953 (82)	0.0267	744 (38)	1209 (62)
Undeveloped	2383 (100)	0.0283	197 (8)	2186 (92)

Weight Matrix Selection

- Based on CLIC statistics: Higher the value, better is the weight matrix specification

	Weight Matrix Specification				
	Contiguous grid	**Shared boundary length**	**Inverse of continuous distance (0.25 mile distance band)**	**Inverse of continuous distance root (0.25 mile distance band)**	**Inverse of continuous distance square (0.25 mile distance band)**
Log-composite likelihood at convergence	-76320.00	-149000.00	-76250.00	-76290.00	-78370.00
Trace Value	628.20	3347.00	530.00	547.50	561.40
CLIC statistics	-76948.20	-152347.00	-76780.00	-76837.50	-78931.40

•All the results are based on inverse of continuous distance weight matrix specification with a distance band of 0.25 miles

Estimation results (mean estimates and t-statistics in parenthesis)

Variables	Spatial multiple discrete continuous probit (SMDCP) model		
	Commercial	**Industrial**	**Residential**
Alternative specific constant *Standard deviation*	-0.488 (-1.15) 0.442 (4.49)	1.283 (2.37) —	-1.715 (-1.79) —
Distance to MoPac (miles)	-0.069 (-4.51)	0.169 (3.03)	-0.063 (-5.47)
Distance to IH-35 (miles) *Standard deviation*	-0.115 (-3.52) —	-0.383 (-5.35) —	0.039 (4.15) 0.118 (4.42)
Distance to US-183 (miles)	—	-0.323 (-7.95)	—
Distance to nearest thoroughfare (miles) *Standard deviation*	-0.325 (-2.27) —	-1.900 (-3.83) 2.883 (6.45)	0.251 (2.888) —
Distance to Hospital (miles)	-0.255 (-7.11)	0.224 (3.44)	0.027 (1.58)
Distance to School (miles)	-0.216 (-3.49)	0.536 (3.33)	-0.455 (-10.51)
Distance to nearest thoroughfare /Distance to floodplain *Standard deviation*	-0.358 (-8.88) 0.246 (2.15)	-0.372 (-2.98) 0.416 (2.13)	0.090 (4.13) 0.165 (6.42)
Fraction of area under floodplain in the grid	-0.015 (-8.92)	-0.022 (-5.41)	-0.010 (-9.70)
Elevation indicator variable (high or low) *Standard deviation*	-0.265 (-4.51) 0.989 (6.57)	-1.429 (-7.74) —	0.180 (3.50) —
CBD indicator variable	—	-1.079 (-2.55)	-0.776 (-6.84)
Satiation parameter	8.873 (19.01)	3.502 (10.56)	44.939 (14.47)
Spatial lag	0.300 (2.36)	0.623 (2.09)	0.477 (4.95)

Effect of Variables on the Utility of Alternative

Variables	Spatial multiple discrete continuous probit (SMDCP) model		
	Commercial	**Industrial**	**Residential**
Alternative specific constant	—	—	—
Distance to MoPac (miles)	positive	negative	positive
Distance to IH-35 (miles)	positive	positive	negative
Distance to US-183 (miles)	—	positive	—
Distance to nearest thoroughfare (miles)	positive	positive	negative
Distance to Hospital (miles)	positive	negative	negative
Distance to School (miles)	positive	negative	positive
Distance to nearest thoroughfare /Distance to floodplain	positive	positive	negative
Fraction of area under floodplain in the grid	negative	negative	negative
Elevation indicator variable (high or low)	negative	negative	positive
CBD indicator variable	—	negative	negative

Error covariance and Model Comparison

Land Use	Commercial	Industrial	Residential
Commercial	1.000 (fixed)	1.445 (4.33)	0.204 (2.30)
Industrial		5.375 (3.22)	0.138 (2.66)
Residential			0.596 (4.94)

Model	Composite Likelihood value	# of parameters estimated	ADCLRT Statistics
MDCP	-76320.00	40	34.72
SMDCP	-76250.00	48	

- The ADCLRT statistics is greater than chi-square critical value at 8 degree's of freedom for any level of significance.

Elasticity Comparison

- We compute the elasticity effect for each of the variable for both MDCP and SMDCP model.

- Continuous variables are increased by 25%

Elasticity Comparison

Scenario	Commercial			Industrial			Residential			Undeveloped		
	MDCP	**SMDCP**	***P*+**	**MDCP**	**SMDCP**	***P***	**MDCP**	**SMDCP**	***P***	**MDCP**	**SMDCP**	***P***
A 25% increase in distance to MoPac	-4.92 (0.61)	-8.10 (1.53)	0.0005	10.99 (0.92)	17.75 (4.94)	0.0005	-4.60 (0.22)	-7.61 (0.62)	0.0005	0.28 (0.17)	0.41 (0.34)	0.0400
A 25% increase in distance to IH35	-2.86 (0.78)	-0.26 (5.49)	0.0240	-10.95 (1.47)	-23.21 (4.16)	0.0005	7.51 (0.37)	11.81 (1.87)	0.0005	0.29 (0.24)	0.41 (0.73)	—*
A 25% increase in distance to US-183	3.48 (0.78)	7.46 (5.44)	0.0030	-8.11 (0.81)	-20.44 (2.66)	0.0005	1.46 (0.24)	3.32 (0.90)	0.0005	0.15 (0.22)	0.38 (0.59)	0.0400
A 25% increase in distance to nearest thoroughfare	-1.31 (0.63)	-3.04 (8.03)	0.1800	4.82 (1.64)	-13.24 (7.02)	0.0005	3.85 (0.27)	9.06 (1.66)	0.0005	0.14 (0.20)	0.06 (0.86)	0.1700
A 25% increase in distance to nearest hospital	-6.97 (0.36)	-11.30 (1.80)	0.0005	11.43 (1.63)	20.29 (5.73)	0.0005	0.97 (0.22)	0.79 (0.67)	0.1300	0.07 (0.13)	0.01 (0.33)	—
A 25% increase in distance to nearest school	-3.88 (0.58)	-6.79 (2.15)	0.0005	5.05 (0.86)	9.62 (2.89)	0.0005	-6.53 (0.22)	-12.70 (0.79)	0.0005	0.60 (0.17)	1.02 (0.40)	0.0005
A 25% increase in distance to nearest thoroughfare and a 25% decrease in distance to floodplain	-5.38 (0.55)	-6.71 (5.83)	0.1700	-0.99 (1.56)	-9.26 (6.15)	0.0005	6.96 (0.34)	12.94 (1.63)	0.0005	0.08 (0.19)	0.23 (0.68)	0.1900
A 25% increase in fraction of area under floodplain in the grid	-1.20 (0.19)	-1.00 (1.11)	—	-1.77 (0.38)	-5.59 (1.16)	0.0005	-1.23 (0.08)	-2.28 (0.36)	0.0005	0.34 (0.07)	0.63 (0.22)	0.0005
A switch of the grid location from lower elevation to higher elevation	36.72 (4.98)	143.40 (19.66)	0.0005	-42.46 (2.72)	-74.83 (2.37)	0.0005	40.88 (1.81)	116.20 (22.67)	0.0005	0.04 (0.87)	1.72 (7.51)	0.1800
A switch of the grid location from non CBD zone to CBD zone	19.05 (1.24)	34.67 (19.75)	0.0005	-64.71 (2.28)	-88.63 (3.04)	0.0005	-50.55 (0.57)	-75.72 (1.80)	0.0005	6.87 (0.53)	10.89 (2.57)	0.0005

Elasticity Comparison Cont..

- General trend : lower elasticity projections from the aspatial model (MDCP), manifestation of neglecting spatial dependencies
- Elasticity effects are opposite in direction for some variable (elasticity effect for industrial land use due to change in distance to nearest thoroughfare)
- Elasticity effects can be misleading, if spatial interactions are neglected.

Conclusion

- Developed a spatial multiple discrete continuous model which accommodates spatial interactions, spatial heterogeneity and error covariance
- MACML estimator is being proposed for estimation as oppose to traditional simulation techniques
- Simulation results shows MACML's excellent capability of recovering parameters irrespective of magnitude of spatial dependency
- Ignoring error covariance, or spatial heterogeneity, or spatial dependency, when present can lead to biased estimation
- Better data fit through spatial model than aspatial model

Thank You

For more details, please visit:

http://www.ce.utexas.edu/prof/bhat/

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